Some schemes for the implementation of implicit Runge-Kutta methods

Cooper, G.J.[a] and Vignesvaran, R.[b]

[a] School of Mathematical and Physical Sciences, University of Sussex, Brighton, United Kingdom
[b] Department of Mathematics, Eastern University, Chenkalady, Sri Lanka

Abstract

The nonlinear equations, arising in the implementation of implicit Runge-Kutta methods, may be solved by a modified Newton iteration, but alternative iteration schemes have been suggested to reduce the linear algebra costs. A linear iteration scheme is examined in this article. When applied to an s-stage Runge-Kutta method, each step of the iteration requires s function evaluations and the solution of s sets of linear equations. For the scalar differential equation $x' = qx$, the convergence rate of the scheme depends on the spectral radius $\pi[M(z)]$ of the iteration matrix M, a function of $z = hq$ where h is the steplength. A lower bound for $\pi[M(z)]$ is established and new schemes are obtained for the two-stage Gauss method by minimizing the supremum of this lower bound over regions of the complex plane. In one scheme the supremum on the negative real axis is minimized. The iteration scheme is generalized in order to obtain improved convergence rates. When applied to an s-stage Runge-Kutta method, each step of this new scheme still requires just s function evaluations. However r sets of linear equations, $r > s$, have to be solved in each step. Some results are obtained for the Gauss methods and some numerical experiments reported.

Author keywords

Implementation; implicit methods; Runge-Kutta