Global optimality principles for polynomial optimization over box or bivalent constraints by separable polynomial approximations

Jeyakumar, V.[a] , Li, G.[a] and Srisatkunarajah, S.[b]

[a] Department of Applied Mathematics, University of New South Wales, Sydney, 2052, Australia
[b] Department of Mathematics and Statistics, University of Jaffna, Jaffna, Sri Lanka

Abstract

In this paper we present necessary conditions for global optimality for polynomial problems with box or bivalent constraints using separable polynomial relaxations. We achieve this by first deriving a numerically checkable characterization of global optimality for separable polynomial problems with box as well as bivalent constraints. Our necessary optimality conditions can be numerically checked by solving semi-definite programming problems. Then, by employing separable polynomial under-estimators, we establish sufficient conditions for global optimality for classes of polynomial optimization problems with box or bivalent constraints. We construct underestimators using the sum of squares convex (SOS-convex) polynomials of real algebraic geometry. An important feature of SOS-convexity that is generally not shared by the standard convexity is that whether a polynomial is SOS-convex or not can be checked by solving a semidefinite programming problem. We illustrate the versatility of our optimality conditions by simple numerical examples.

Author keywords

Bivalent constraints; Box constraints; Global optimality conditions; Global optimization; Polynomial optimization