Lagrange multiplier necessary conditions for global optimality for non-convex minimization over a quadratic constraint via S-lemma

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Abstract

In this paper, we present Lagrange multiplier necessary conditions for global optimality that apply to non-convex optimization problems beyond quadratic optimization problems subject to a single quadratic constraint. In particular, we show that our optimality conditions apply to problems where the objective function is the difference of quadratic and convex functions over a quadratic constraint, and to certain class of fractional programming problems. Our necessary conditions become necessary and sufficient conditions for global optimality for quadratic minimization subject to quadratic constraint. As an application, we also obtain global optimality conditions for a class of trust-region problems. Our approach makes use of outer-estimators, and the powerful S-lemma which has played key role in control theory and semidefinite optimization. We discuss numerical examples to illustrate the significance of our optimality conditions.

Author keywords

Difference of quadratic and convex functions; Fractional programs; Global optimality; Lagrange multipliers; Single quadratic constraint; Smooth non-convex minimization

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