

Phase Space Stability Error Control with Variable Time-stepping Runge-Kutta Methods for Dynamical Systems

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Abstract

We consider a phase space stability error control for numerical simulation of dynamical systems. We illustrate how variable time-stepping algorithms perform poorly for long time computations which pass close to a fixed point. A new error control was introduced in [9], which is a generalization of the error control first proposed in [8]. In this error control, the local truncation error at each step is bounded by a fraction of the solution arc length over the corresponding time interval. We show how this error control can be thought of either a phase space or a stability error control. For linear systems with a stable hyperbolic fixed point, this error control gives a numerical solution which is forced to converge to the fixed point. In particular, we analyze the forward Euler method applied to the linear system whose coefficient matrix has real and negative eigenvalues. We also consider the dynamics in the neighborhood of saddle points. We introduce a step-size selection scheme which allows this error control to be incorporated within the standard adaptive algorithm as an extra constraint at negligible extra computational cost. Theoretical and numerical results are presented to illustrate the behavior of this error control. (© 2004 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim)