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Solitary wave solutions of the Camassa–Holm-Nonlinear Schrödinger Equation

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ABSTRACT

This study investigates the solitary wave solutions to the defocusing nonlinear Schrödinger equation, which is known as Camassa–Holm-Nonlinear Schrödinger (CH-NLS) equation. The CH-NLS equation is newly derived in the sense of deformation of hierarchies structure of integrable systems. By implementing three different techniques, namely, the generalized (G'/G)-expansion method, the new mapping method, and the modified simple equation method, the CH-NLS equation is solved analytically to find the exact solutions. As a result, various types of solitons such as dark, singular, and periodic solutions are obtained. The behaviors of some exact solutions are presented by figures.

1. Introduction

The study on deformations of integrable equations has been a great interest since it produces many significant models. For example, the deformation structure of the Korteweg–de Vries (KdV) equation leads to the Camassa–Holm (CH) equation as

$$m_t + 2mu_x + m_x u = 0, \qquad m = u - a^2 u_{xx}, \tag{1}$$

which is an integrable equation originally derived in the sense of water waves [1]. This model has been considered as much interest since its soliton solutions [2] and some mathematical properties [3,4]. Consequently, many authors started to find the solitary wave solutions of CH-equation by using various techniques [5–7].

Very recently, the deformation of nonlinear Schrödinger equation, i.e., the CH-NLS equation was derived by Arnaudon [8,9] when extending the structure of deformation of hierarchies of integrable method. The form of the CH-NLS equation as follows:

$$im_t + u_{xx} + 2\sigma m(|u|^2 - a^2|u_x|^2) = 0, \qquad m = u - a^2 u_{xx}, \tag{2}$$

were, $\sigma = \pm 1$, and a is a constant. For instant, if we choose $a = 0$ the above equation becomes the well known cubic NLS equation as follows:

$$iu_t + u_{xx} + 2\sigma|u|^2 u = 0. \tag{3}$$

The standard form of CH-NLS equation is given for a complex field $u(x,t)$ by [10]:

$$iu_t + u_{xx} + 2\sigma u|u|^2 - ia^2 u_{xxt} - 2\sigma a^2 u|u_x|^2 - 2\sigma a^2 u_{xx}|u|^2 + 2\sigma a^4 u_{xx}|u_x|^2 = 0. \tag{4}$$

In [10], asymptotic multiscale expansion method is applied to reduce Eq. (4) to a Boussinesq model and then approximate by the couple of Korteweg–de Vries (KdV) equations. The solitary wave solution of the famous KdV equation is used to find approximate analytical solutions of the CH-NLS equation. Further, numerical simulations are used to study the validity of the approximate solutions and illustrate their dynamical evolution. However, as far as we know, there is no study involved to find the exact solution of the CH-NLS equation. The central aim of the present study is to find the exact solitons and other kinds of solutions to the CH-NLS equation. In generally, nonlinear Schrödinger equation provides two different types of soliton solutions, namely bright soliton and dark soliton. In the context of nonlinear optics, dark solitons are constructed as topological optical solitons. In recent years, studies of the optical solitons have been increasing rapidly [11–14] due to the various applications in communication, imaging, medical diagnosis, etc.

Finding the explicit and exact solutions to the nonlinear type of partial differential equations (PDEs) are significant role in many area of Mathematical physics. In recent years, several new technique for finding these exact solutions have been implemented, for example, the tanh–sech method [15,16], the sine–cosine method, [17,18], the homogeneous balance method [19], the Decomposition method [20,21] the Jacobi elliptic function method [22,23], the F-expansion method [24, 25], the exp-function method [26,27], the (G'/G) expansion method [28–31], the new approach of generalized (G'/G) expansion method [32, 33], the new mapping method [34,35], the modified simple equation

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method [36–39], the Hirota method [40,41], the Darboux transformation method [42,43], the Hirota bilinear method [44,45] and so on.

The paper aims to apply the new approach of generalized (G'/G)-expansion method, the new mapping method, and the modified simple equation method to find the exact soliton solution of CH-NLS equation. This manuscript is structured as follows: In the next Section, we describe the mathematical analysis of Eq. (4). In Sections 3–5, Eq. (4) is solved by using three various technique as mentioned above. The graphs of solitary wave solutions are presented in Section 6. Finally, concluding remarks are given in Section 7.

2. Mathematical analysis of the model

As indicated above, our central aim is to determine exact soliton solutions of the CH-NLS equation. Since $u(x,t)$ in Eq. (4) is a complex-valued function we consider the traveling wave solution of the form

$$u(x,t) = \phi(\xi)e^{I(Kx-\Omega t)} \quad \xi = \omega x - ct, \tag{5}$$

where $\phi(\xi)$ is a real function, K, Ω, ω and c are real constants to be determined later. By substituting (5) into Eq. (4) and splitting the real and imaginary parts, we have the following couple nonlinear ordinary differential equations:

$$\begin{aligned}&\phi(\xi)\left(\Omega + K^2\left(-1+a^2\Omega\right) - 2\left(-1+a^4K^4\right)\sigma\phi(\xi)^2\right.\\ &\quad -2a^2\left(1+a^2K^2\right)\sigma\omega^2\phi'(\xi)^2\big) + \\ &\quad \omega\left(\omega - a^2(2cK+\omega\Omega) + 2a^2\left(-1+a^2K^2\right)\sigma\omega\phi(\xi)^2\right.\\ &\quad \left.+2a^4\sigma\omega^3\phi'(\xi)^2\right)\phi''(\xi) = 0,\end{aligned} \tag{6}$$

and

$$\begin{aligned}&-\left(c + a^2cK^2 + 2K\omega\left(-1+a^2\Omega\right) - 4a^2K\left(-1+a^2K^2\right)\sigma\omega\phi(\xi)^2\right)\phi'(\xi) + \\ &\qquad 4a^4K\sigma\omega^3\phi'(\xi)^3 + a^2c\omega^2\phi^{(3)}(\xi) = 0,\end{aligned} \tag{7}$$

where prime meaning the differentiation with respect to ξ. Multiplying Eqs. (6) and (7) by $\phi'(\xi)$ and $\phi(\xi)$ respectively and adding together, we get:

$$\begin{aligned}&2\sigma\left(1 - a^4K^3(K-2\omega) - 2a^2K\omega\right)\phi(\xi)^3\phi'(\xi)\\ &+ 2a^2\left(a^2K^2-1\right)\sigma\omega^2\phi(\xi)^2\phi'(\xi)\phi''(\xi) + \\ &\omega\ \left(\omega - a^2(2cK+\omega\Omega)\right)\phi'(\xi)\phi''(\xi) + 2a^4\sigma\omega^4\phi'(\xi)^3\phi''(\xi)\\ &-2a^2\sigma\left(1+a^2K(K-2\omega)\right)\omega^2\phi(\xi)\phi'(\xi)^3\\ &-\left(c + a^2c\omega^2 - \Omega + \omega^2\left(a^2\Omega - 1\right)\right)\phi(\xi)\phi'(\xi) + a^2c\omega^2\phi(\xi)\phi^{(3)}(\xi) = 0\end{aligned} \tag{8}$$

Integrating Eq. (8) once, with respect to ξ and considering $K = \omega$, we get

$$b_0\phi(\xi)\phi''(\xi) + b_1\phi(\xi)^2 + b_2\phi(\xi)^4 + b_3\phi'(\xi)^2 + b_4\phi(\xi)^2\phi'(\xi)^2 + b_5\phi'(\xi)^4 + b_6 = 0, \tag{9}$$

where the constants b_0, b_1, b_2, b_3, b_4, and b_5 are given by

$$\begin{aligned}b_0 &= a^2c\omega^2,\\ b_1 &= -\frac{1}{2}\left(c + a^2c\omega^2 - \Omega + \omega^2\left(a^2\Omega - 1\right)\right),\\ b_2 &= \frac{1}{2}\sigma\left(a^2\omega^2 - 1\right)^2,\\ b_3 &= -\frac{1}{2}\omega^2\left(a^2(3c+\Omega) - 1\right),\\ b_4 &= a^2\sigma\omega^2\left(a^2\omega^2 - 1\right),\\ b_5 &= \frac{1}{2}a^4\sigma\omega^4,\end{aligned}$$

and b_6 is integrating constant. First, taking the homogeneous balance between $\phi(\xi)\phi''(\xi)$ and $\phi'(\xi)^4$ yields $N = -1$, which is not a positive integer. Following [16], we consider the transformation

$$\phi(\xi) = \frac{1}{\psi(\xi)}, \tag{10}$$

where $\psi(\xi)$ is a real function on ξ. By Substituting (10) into (9), we obtain a new equation as follows

$$\begin{aligned}&-b_0\psi(\xi)^5\psi''(\xi) + b_1\psi(\xi)^6 + b_2\psi(\xi)^4 + (b_3 + 2b_0)\psi(\xi)^4\psi'(\xi)^2\\ &\quad + b_4\psi(\xi)^2\psi'(\xi)^2 + b_5\psi'(\xi)^4 + b_6\psi(\xi)^8 = 0.\end{aligned} \tag{11}$$

Now, our aim is to solve Eq. (11) for $a \neq 0$ using three different technique as mentioned above.

3. Soliton solutions of CH-NLS equation through the generalized (G'/G)-expansion method

Taking the homogeneous balance of $\psi(\xi)^5\psi''(\xi)$ and $\psi(\xi)^8$ in Eq. (11), we obtain $N = 1$. According to the new approach of generalized (G'/G) expansion method [32,33], the solution of Eq. (11) is of the form:

$$\psi(\xi) = \alpha_{-1}(d + G'/G)^{-1} + \alpha_0 + \alpha_1(d + G'/G), \tag{12}$$

where the constants $\alpha_{-1}, \alpha_0, \alpha_1$ and d are to be determined later and $G(\xi)$ satisfied the following equation:

$$\kappa G(\xi)G''(\xi) - \lambda G(\xi)G'(\xi) - \mu G(\xi)^2 - \nu G'(\xi)^2 = 0. \tag{13}$$

Substitute Eq. (12) together with Eq. (13) into Eq. (11). Then, we take each coefficient of $(G'/G)^N$ and setting them to zero, yields a system of algebraic equations for the possible choice of $\alpha_{-1} = \alpha_0 = \lambda = 0$ as follows:

$$\begin{aligned}
&[G'/G]^0: && \mu + d^2(-\kappa+\nu) = 0,\\
&[G'/G]^1: && 8d\left(-\mu + d^2(\kappa-\nu)\right)^3(\kappa-\nu)b_5\alpha_1^4 = 0,\\
&[G'/G]^2: && \left(\mu + d^2(-\kappa+\nu)\right)^2\left(\kappa^2b_4 - 4\left(\mu - 7d^2(\kappa-\nu)\right)(\kappa-\nu)b_5\right)\alpha_1^4 = 0,\\
&[G'/G]^3: && 4d\left(-\mu + d^2(\kappa-\nu)\right)(\kappa-\nu)\left(\kappa^2b_4 + 2\left(-3\mu\right.\right.\\
& && \quad \left.\left.+7d^2(\kappa-\nu)\right)(\kappa-\nu)b_5\right)\alpha_1^4 = 0,\\
&[G'/G]^4: && \alpha_1^4\left(\kappa^4b_2 + 2(\kappa-\nu)\left(\kappa^2\left(-\mu + 3d^2(\kappa-\nu)\right)b_4\right.\right.\\
& && \quad + (\kappa-\nu)\left(3\mu^2 + 35d^4(\kappa-\nu)^2\right.\\
& && \quad \left.\left.+30d^2\mu(-\kappa+\nu)\right)b_5\right) + \kappa^2\left(\mu + d^2(-\kappa+\nu)\right)^2\left(2b_0 + b_3\right)\alpha_1^2\Big) = 0,\\
&[G'/G]^5: && 2d(\kappa-\nu)\alpha_1^4\left(2\kappa^2(\kappa-\nu)b_4 + 4\left(-3\mu + 7d^2(\kappa-\nu)\right)(\kappa-\nu)^2b_5\right.\\
& && \quad \left.+\kappa^2\left(-\mu + d^2(\kappa-\nu)\right)\left(3b_0 + 2b_3\right)\alpha_1^2\right) = 0,\\
&[G'/G]^6: && \alpha_1^4\left(\kappa^2(\kappa-\nu)^2b_4 - 4\left(\mu - 7d^2(\kappa-\nu)\right)(\kappa-\nu)^3b_5\right.\\
& && \quad + \kappa^2\left(-2\left(\mu - 3d^2(\kappa-\nu)\right)(\kappa-\nu)b_0\right.\\
& && \quad \left.\left.+\kappa^2b_1 - 2\left(\mu - 3d^2(\kappa-\nu)\right)(\kappa-\nu)b_3\right)\alpha_1^2\right) = 0,\\
&[G'/G]^7: && 2d(\kappa-\nu)^2\alpha_1^4\left(4(\kappa-\nu)^2b_5 + \kappa^2\left(b_0 + 2b_3\right)\alpha_1^2\right) = 0,\\
&[G'/G]^8: && \alpha_1^4\left((\kappa-\nu)^4b_5 + \kappa^2\alpha_1^2\left((\kappa-\nu)^2b_3 + \kappa^2b_6\alpha_1^2\right)\right) = 0.
\end{aligned} \tag{14}$$

By solving the above algebraic equations with the use of Mathematica software, we have the following result:

$$\begin{aligned}&\mu = -d^2(-\kappa+\nu),\ \alpha_1 = \frac{2(\kappa-\nu)}{\kappa}\sqrt{\frac{b_5}{(-b_0-2b_3)}},\ d = \frac{\kappa}{2(\kappa-\nu)}\sqrt{-\frac{b_4}{2b_5}},\\ &\qquad b_1 = -\frac{b_3b_4}{2b_5},\ b_2 = \frac{b_4^2}{4b_5},\ b_6 = \frac{-b_0^2+4b_3^2}{16b_5},\end{aligned} \tag{15}$$

By substituting Eq. (15) into Eq. (12), together with Eq. (10), we obtain

$$\phi(\xi) = \left\{\frac{2(\kappa-\nu)}{\kappa}\sqrt{\frac{b_5}{(-b_0-2b_3)}}\left[\frac{\kappa}{2(\kappa-\nu)}\sqrt{-\frac{b_4}{2b_5}} + (G'/G)\right]\right\}^{-1}. \tag{16}$$

With reference to [32,33], two types of traveling wave solutions are obtained of the Eq. (2) as follows:

Type 1. If $\lambda = 0, (\kappa-\nu)\mu > 0 \Rightarrow -\frac{b_4}{2b_5} > 0$, then we obtain the hyperbolic solution of Eq. (2):

$$u(x,t) = \left\{ \sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}} \times \left[1 + \frac{c_1 \sinh\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{-\frac{b_4}{2b_5}}\frac{\xi}{2}\right) + c_2 \cosh\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{-\frac{b_4}{2b_5}}\frac{\xi}{2}\right)}{c_1 \cosh\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{-\frac{b_4}{2b_5}}\frac{\xi}{2}\right) + c_2 \sinh\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{-\frac{b_4}{2b_5}}\frac{\xi}{2}\right)} \right] \right\}^{-1} e^{I(\omega x - \Omega t)}, \tag{17}$$

where $\xi = \omega x - \left(\frac{1}{2a^2} + \frac{\Omega - a^2\omega^2\Omega}{-1+2a^2\omega^2}\right)t$, and c_1 and c_2 are constants.

In particular, if we choose "$c_1 \neq 0, c_2 = 0$" in (17), then we obtain the dark solitary wave solutions to Eq. (2) as:

$$u(x,t) = \left\{ \sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}} \left[1 + \tanh\left(\frac{\kappa}{(\kappa-\nu)} \sqrt{-\frac{b_4}{2b_5}} \frac{\xi}{2} \right) \right] \right\}^{-1} e^{I(\omega x - \Omega t)}. \tag{18}$$

If we choose "$c_1 = 0, c_2 \neq 0$" in (17), then we obtain the singular solitary wave solution to Eq. (2) as:

$$u(x,t) = \left\{ \sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}} \left[1 + \coth\left(\frac{\kappa}{(\kappa-\nu)} \sqrt{-\frac{b_4}{2b_5}} \frac{\xi}{2} \right) \right] \right\}^{-1} e^{I(\omega x - \Omega t)}. \tag{19}$$

Type 2. If $\lambda = 0, (\kappa-\nu)\mu < 0 \Rightarrow -\frac{b_4}{2b_5} < 0$, we find the trigonometric solution of Eq. (2) as follows:

$$u(x,t) = \left\{ \sqrt{-\frac{b_4}{2\left(b_0+2b_3\right)}} \times \left[1 + \frac{c_1 \sin\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{\frac{b_4}{2b_5}}\frac{\xi}{2}\right) + c_2 \cos\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{\frac{b_4}{2b_5}}\frac{\xi}{2}\right)}{c_1 \cos\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{\frac{b_4}{2b_5}}\frac{\xi}{2}\right) + c_2 \sin\left(\frac{\kappa}{(\kappa-\nu)}\sqrt{\frac{b_4}{2b_5}}\frac{\xi}{2}\right)} \right] \right\}^{-1} e^{I(\omega x - \Omega t)}, \tag{20}$$

where $\xi = \omega x - \left(\frac{1}{2a^2} + \frac{\Omega - a^2\omega^2\Omega}{-1+2a^2\omega^2}\right)t$, and c_1 and c_2 are constants.

Again, for a specific choice, if we set "$c_1 \neq 0, c_2 = 0$" or "$c_1 = 0, c_2 \neq 0$" in (20), then we obtain the periodic solitary wave solutions to Eq. (2) as:

$$u(x,t) = \left\{ \sqrt{-\frac{b_4}{2\left(b_0+2b_3\right)}} \left[1 + \tan\left(\frac{\kappa}{(\kappa-\nu)} \sqrt{\frac{b_4}{2b_5}} \frac{\xi}{2} \right) \right] \right\}^{-1} e^{I(\omega x - \Omega t)}, \tag{21}$$

$$u(x,t) = \left\{ \sqrt{-\frac{b_4}{2\left(b_0+2b_3\right)}} \left[1 + \cot\left(\frac{\kappa}{(\kappa-\nu)} \sqrt{\frac{b_4}{2b_5}} \frac{\xi}{2} \right) \right] \right\}^{-1} e^{I(\omega x - \Omega t)}. \tag{22}$$

4. Soliton solutions of CH-NLS equation through the new mapping method

Balancing $\psi(\xi)^2\psi'(\xi)^2$ with $\psi(\xi)^5\psi''(\xi)$ in Eq. (11), give up the homogeneous balance number $N = 1$. Based on the new mapping method [34,35], we take the solution of form:

$$\psi(\xi) = \alpha_0 + \alpha_1 F(\xi) + \alpha_2 F^2(\xi), \tag{23}$$

where the constants α_0, α_1 and $\alpha_2 \neq 0$ are to be determined later. The new function $F(\xi)$ satisfies the following ODE:

$$F'^2(\xi) = r + pF^2(\xi) + \frac{q}{2}F^4(\xi) + \frac{s}{3}F^6(\xi), \tag{24}$$

here r, p, q, and $s \neq 0$ are constants. By substituting (23) together with (24) into Eq. (4) and take the coefficients of each power $F^i(\xi)(F'(\xi))^j$. By setting these coefficients to be zero, we have a set of algebraic equations for the possible choice of $\alpha_0 = \alpha_1 = 0$ as follows:

$$\begin{aligned}
F^4(\xi):&\quad 16r^2 b_5 \alpha_2^4 = 0,\\
F^6(\xi):&\quad 4r\left(b_4 + 8pb_5\right)\alpha_2^4 = 0,\\
F^8(\xi):&\quad \left(b_2 + 4\left(pb_4 + 4\left(p^2 + qr\right)b_5\right)\right)\alpha_2^4 = 0,\\
F^{10}(\xi):&\quad \frac{2}{3}\alpha_2^4\left(3qb_4 + 8(3pq + 2rs)b_5 + 3r\left(3b_0 + 2b_3\right)\alpha_2^2\right) = 0,\\
F^{12}(\xi):&\quad \frac{1}{3}\alpha_2^4\left(4sb_4 + 4\left(3q^2 + 8ps\right)b_5 + 3\left(4pb_0 + b_1 + 4pb_3\right)\alpha_2^2\right) = 0,\\
F^{14}(\xi):&\quad \frac{16}{3}qsb_5\alpha_2^4 + q\left(b_0 + 2b_3\right)\alpha_2^6 = 0,\\
F^{16}(\xi):&\quad \frac{16}{9}s^2 b_5 \alpha_2^4 + \frac{4}{3}sb_3\alpha_2^6 + b_6\alpha_2^8 = 0,
\end{aligned} \tag{25}$$

and the other coefficients of powers of $F^i(\xi)$ are equal to zero. According to Ref. [34,35], we have two types of solutions of the above algebraic equations as follows:

Type 1. Considering $r = 0,\ s = \frac{3q^2}{16p}$ and solving the algebraic equations by Mathematica, we obtain the following result:

$$\alpha_2 = \frac{2\sqrt{2}qb_5}{\sqrt{\left(b_0+2b_3\right)b_4}},\quad p = -\frac{b_4}{8b_5},\quad b_1 = -\frac{b_3 b_4}{2b_5},\quad b_2 = \frac{b_4^2}{4b_5},\quad b_6 = \frac{-b_0^2 + 4b_3^2}{16b_5}. \tag{26}$$

From (26), (23), and (10) with Ref. [34,35], we find the soliton solutions of Eq. (2) as follows:

$$u(x,t) = \left\{ \sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}} \left[1 + \tanh\left(\epsilon\sqrt{-\frac{b_4}{8b_5}}\,\xi \right) \right] \right\}^{-1} e^{I(\omega x - \Omega t)}, \tag{27}$$

$$u(x,t) = \left\{ \sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}} \left[1 + \coth\left(\epsilon\sqrt{-\frac{b_4}{8b_5}}\,\xi \right) \right] \right\}^{-1} e^{I(\omega x - \Omega t)}, \tag{28}$$

provided that if $p = -\frac{b_4}{8b_5} > 0$.

Type 2. Considering $r = 0$ and solving the algebraic equations by using Mathematica, we obtain the following result:

$$\alpha_2 = \frac{2\sqrt{2}qb_5}{\sqrt{\left(b_0+2b_3\right)b_4}},\quad p = -\frac{b_4}{8b_5},\quad s = -\frac{3q^2 b_5}{2b_4},$$
$$b_1 = -\frac{b_3 b_4}{2b_5},\quad b_2 = \frac{b_4^2}{4b_5},\quad b_6 = \frac{-b_0^2 + 4b_3^2}{16b_5}. \tag{29}$$

From (29), (23), and (10) with Ref. [34,35], we find the exact solutions of Eq. (2) as follows:

1. If $p = -\frac{b_4}{8b_5} > 0$, then we obtain soliton solutions

$$u(x,t) = \left\{ 3q^2\sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}} \times \left[\frac{2\,\mathrm{sech}^2\left(\sqrt{-\frac{b_4}{8b_5}}\,\xi\right) b_5}{6q^2 b_5 + sb_4\left(1 + \epsilon\tanh\left(\sqrt{-\frac{b_4}{8b_5}}\,\xi\right)\right)^2} \right] \right\}^{-1} e^{I(\omega x - \Omega t)} \tag{30}$$

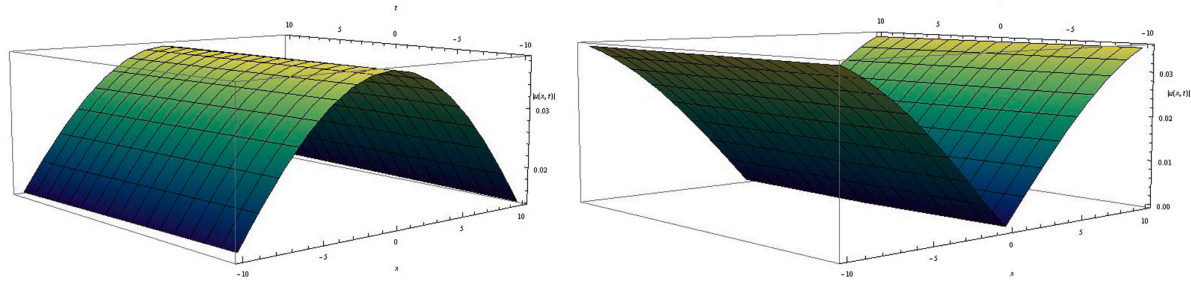

Fig. 1. The graphics of solitary wave solutions $u(x,t)$ of Eqs. (18) and (19) with $a = 2,\ \sigma = 1,\ \omega = 1,\ \Omega = 0.1$..

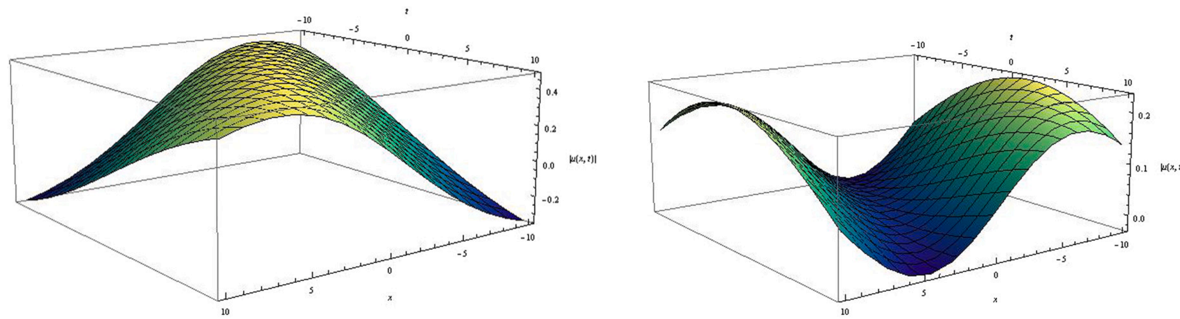

Fig. 2. The graphics of solitary wave solutions $u(x,t)$ of Eqs. (21) and (22) with $a = 2,\ \sigma = 1,\ \omega = 0.1,\ \Omega = 0.1$..

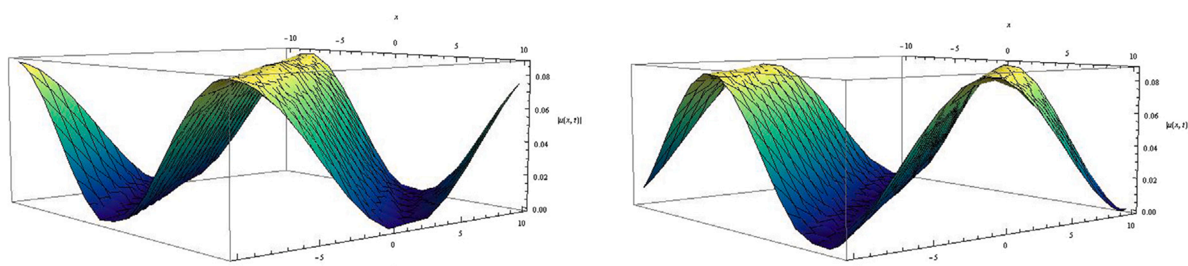

Fig. 3. The graphics of soliton wave solutions $u(x,t)$ of Eqs. (34) and (35) with $a = 4,\ \sigma = 1,\ \omega = 0.4,\ \Omega = 1$..

$$\mathrm{u}(x,t) = \left\{ 3q^2 \sqrt{\frac{b_4}{2\left(b_0 + 2b_3\right)}} \times \left[\frac{2\operatorname{csch}^2\left(\sqrt{-\frac{b_4}{8b_5}}\ \xi\right) b_5}{6q^2 b_5 + s b_4 \left(1 + \epsilon \coth\left(\sqrt{-\frac{b_4}{8b_5}}\ \xi\right)\right)^2} \right] \right\}^{-1} e^{I(\omega x - \Omega t)} \tag{31}$$

2. If $p = -\frac{b_4}{8b_5} > 0,\ s > 0$ then we obtain soliton solutions

$$\mathrm{u}(x,t) = \left\{ 3q \sqrt{\frac{b_4}{2\left(b_0 + 2b_3\right)}} \times \left[\frac{2\operatorname{sech}^2\left(\sqrt{-\frac{b_4}{8b_5}}\ \xi\right) b_5}{3q + \epsilon\sqrt{-\frac{6sb_4}{b_5}}\ \tanh\left(\sqrt{-\frac{b_4}{8b_5}}\ \xi\right)} \right] \right\}^{-1} e^{I(\omega x - \Omega t)} \tag{32}$$

$$\mathrm{u}(x,t) = -\left\{ 3q \sqrt{\frac{b_4}{2\left(b_0 + 2b_3\right)}} \times \left[\frac{2\operatorname{csch}^2\left(\sqrt{-\frac{b_4}{8b_5}}\ \xi\right) b_5}{3q + \epsilon\sqrt{-\frac{6sb_4}{b_5}}\ \coth\left(\sqrt{-\frac{b_4}{8b_5}}\ \xi\right)} \right] \right\}^{-1} e^{I(\omega x - \Omega t)} \tag{33}$$

3. If $p = -\frac{b_4}{8b_5} < 0,\ s > 0$ then we obtain periodic solutions

$$\mathrm{u}(x,t) = \left\{ 3q \sqrt{\frac{b_4}{2\left(b_0 + 2b_3\right)}} \times \left[\frac{2\sec\left(\sqrt{\frac{b_4}{8b_5}}\ \xi\right)^2 b_5}{3q + \epsilon\sqrt{\frac{6sb_4}{b_5}}\ \tan\left(\sqrt{\frac{b_4}{8b_5}}\ \xi\right)} \right] \right\}^{-1} e^{I(\omega x - \Omega t)} \tag{34}$$

$$u(x,t)=\left\{3q\sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}}\times\left[\frac{2\csc\left(\sqrt{\frac{b_4}{8b_5}}\ \xi\right)^2 b_5}{3q+\epsilon\sqrt{\frac{6sb_4}{b_5}}\ \cot\left(\sqrt{\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{35}$$

4. If $p=-\frac{b_4}{8b_5}>0,\ q<0,\ s<0,\ M>0$ then we obtain soliton solution

$$u(x,t)=\left\{-3q\sqrt{\frac{2b_4}{b_0+2b_3}}\left[\frac{1}{-3q+\sqrt{M}\ \mathrm{Cosh}\left(2\epsilon\ \sqrt{-\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{36}$$

5. If $p=-\frac{b_4}{8b_5}<0,\ q>0,\ s<0,\ M>0$ then we obtain soliton solution

$$u(x,t)=\left\{3q\sqrt{\frac{2b_4}{b_0+2b_3}}\left[\frac{1}{3q+\sqrt{M}\ \cosh\left(2\epsilon\ \sqrt{\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{37}$$

6. If $p=-\frac{b_4}{8b_5}>0,\ M>0$, then we obtain soliton solutions

$$u(x,t)=\left\{-3q\sqrt{\frac{2b_4}{b_0+2b_3}}\left[\frac{1}{-3q+\epsilon\sqrt{M}\ \cosh\left(2\sqrt{-\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{38}$$

$$u(x,t)=\left\{-3q\sqrt{\frac{2b_4}{b_0+2b_3}}\left[\frac{1}{-3q+\epsilon\sqrt{-M}\ \sinh\left(2\sqrt{-\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{39}$$

7. If $p=-\frac{b_4}{8b_5}<0,\ M>0$, then we obtain soliton solutions

$$u(x,t)=\left\{-3q\sqrt{\frac{2b_4}{b_0+2b_3}}\left[\frac{1}{-3q+\epsilon\sqrt{M}\ \cos\left(2\sqrt{-\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{40}$$

$$u(x,t)=\left\{-3q\sqrt{\frac{2b_4}{b_0+2b_3}}\left[\frac{1}{-3q+\epsilon\sqrt{M}\ \sin\left(2\sqrt{\frac{b_4}{8b_5}}\ \xi\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)} \tag{41}$$

where $M=9q^2-48ps,\ \ \epsilon=\pm 1$.

5. Soliton solutions of CH-NLS equation through the modified simple equation method

Taking the homogeneous balance between $\psi(\xi)^5\psi''(\xi)$ and $\psi(\xi)^8$ in Eq. (11), we obtain $N=1$. Based on the modified simple equation method [36,37], we consider the following solution:

$$\psi(\xi)=A_0+A_1\left[\frac{\Psi'(\xi)}{\Psi(\xi)}\right], \tag{42}$$

where the constants A_0 and $A_1\neq 0$ are to be determined later. By Substitute (42) into (11), taking all the coefficients of powers of $\Psi(\xi)^{-j}$ and making them to zero, yields a set of algebraic equations for the possible choice of $A_0=0$ as follows:

$$\begin{aligned}
\Psi(\xi)^{-3}:&\quad A_1^4\left(b_2\Psi'(\xi)^4+b_4\Psi'(\xi)^2\Psi''(\xi)^2+b_5\Psi''(\xi)^4\right)=0,\\
\Psi(\xi)^{-4}:&\quad -2A_1^4\Psi'(\xi)^2\Psi''(\xi)\left(b_4\Psi'(\xi)^2+2b_5\Psi''(\xi)^2\right)=0,\\
\Psi(\xi)^{-5}:&\quad A_1^4\Psi'(\xi)^4\left(b_4\Psi'(\xi)^2+6b_5\Psi''(\xi)^2+A_1^2\left(b_1\Psi'(\xi)^2\right.\right.\\
&\quad\left.\left.+\left(2b_0+b_3\right)\Psi''(\xi)^2-b_0\Psi'(\xi)\Psi'''(\xi)\right)\right)=0,\\
\Psi(\xi)^{-6}:&\quad -A_1^4\left(A_1^2\left(b_0+2b_3\right)+4b_5\right)\Psi'(\xi)^6\Psi''(\xi)=0,\\
\Psi(\xi)^{-7}:&\quad A_1^4\left(A_1^2b_3+b_5+A_1^4b_6\right)\Psi'(\xi)^8=0,
\end{aligned} \tag{43}$$

and the other coefficients of powers of $\Psi(\xi)^{-j}$ are equal to zero. Solving last two equations in (43) by Mathematica, we obtain,

$$A_1=2\sqrt{\frac{b_5}{-b_0-2b_3}},\ b_6=\frac{-b_0^2+4b_3^2}{16b_5}. \tag{44}$$

In this case, first three algebraic equations reduce to

$$\begin{aligned}
&b_2\Psi'(\xi)^4+b_4\Psi'(\xi)^2\Psi''(\xi)^2+b_5\Psi''(\xi)^4=0,\\
&b_4\Psi'(\xi)^2+2b_5\Psi''(\xi)^2=0,\\
&2\left(b_3b_4-2b_1b_5\right)\Psi'(\xi)^2+8b_3b_5\Psi''(\xi)^2+b_0\left(b_4\Psi'(\xi)^2\right.\\
&\left.-2b_5\left(\Psi''(\xi)^2-2\Psi'(\xi)\Psi'''(\xi)\right)\right)=0.
\end{aligned} \tag{45}$$

From the system of Eq. (45), we obtain the following results

$$b_1=-\frac{b_3b_4}{2b_5},\ b_2=\frac{b_4^2}{4b_5}. \tag{46}$$

$$\Psi^{(3)}(\xi)+\frac{b_4}{2b_5}\ \Psi'(\xi)=0. \tag{47}$$

Consequently, Eq. (47) reduces to

$$\Psi(\xi)=\sqrt{-\frac{2b_5}{b_4}}\ e^{-\sqrt{-\frac{b_4}{2b_5}}\xi}\left(c_1-c_2e^{\sqrt{-\frac{2b_4}{b_5}}\xi}\right)+c_3, \tag{48}$$

where $\frac{b_4}{b_5}<0$ and c_1,c_2 and c_3 are integration constants. Substituting (48) into (42) along with (10), we obtain the exact solution of Eq. (2) as:

$$u(x,t)=\left\{\sqrt{\frac{2b_4}{\left(b_0+2b_3\right)}}\left[\frac{\left(c_1e^{\sqrt{-\frac{b_4}{2b_5}}\xi}+c_2e^{\sqrt{-\frac{b_4}{2b_5}}\xi}\right)}{\left(-c_1e^{\sqrt{-\frac{b_4}{2b_5}}\xi}+c_2e^{\sqrt{-\frac{b_4}{2b_5}}\xi}+c_3\sqrt{-\frac{b_4}{2b_5}}\right)}\right]\right\}^{-1} e^{I(\omega x-\Omega t)}. \tag{49}$$

As a specific choice, if we set $c_1=0$ and $\frac{c_3}{c_2}=\sqrt{-\frac{2b_5}{b_4}}$ in (49), then we have the dark soliton solution

$$u(x,t)=\left\{\sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}}\left[1+\tanh\left(\sqrt{-\frac{b_4}{2b_5}}\frac{\xi}{2}\right)\right]\right\}^{-1} e^{I(\omega x-\Omega t)}, \tag{50}$$

while, if we set $c_1=0$ and $\frac{c_3}{c_2}=-\sqrt{-\frac{2b_5}{b_4}}$ in (49), then we have the singular soliton solution

$$u(x,t)=\left\{\sqrt{\frac{b_4}{2\left(b_0+2b_3\right)}}\left[1+\coth\left(\sqrt{-\frac{b_4}{2b_5}}\frac{\xi}{2}\right)\right]\right\}^{-1} e^{I(\omega x-\Omega t)}. \tag{51}$$

Note that the above soliton solutions (50) and (51) are consistent with our previous solutions (27) and (28), when $\epsilon=1$, respectively.

6. Graphical representations of the solutions

In this section, we present the physical interpretation of the obtained exact traveling wave solutions of the CH-NLS equation. The graphical illustrations of some solutions are given in Figs. 1–5 for special values of the free parameters. From the above figures, one can see that the obtained solutions possess the dark soliton solutions, the singular soliton solutions, the solitary wave solutions and the periodic wave solutions. Fig. 1 of the solutions (18) and (19) shows the shape

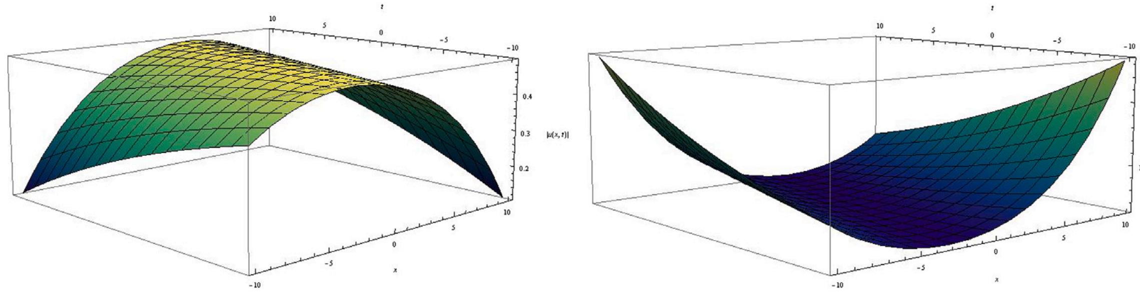

Fig. 4. The graphics of solitary wave solutions $u(x,t)$ of Eqs. (38) and (39) with $a = 3,\ \sigma = 1,\ \omega = 0.3,\ \Omega = 0.1, \epsilon = 1, q = 1, s = 1$..

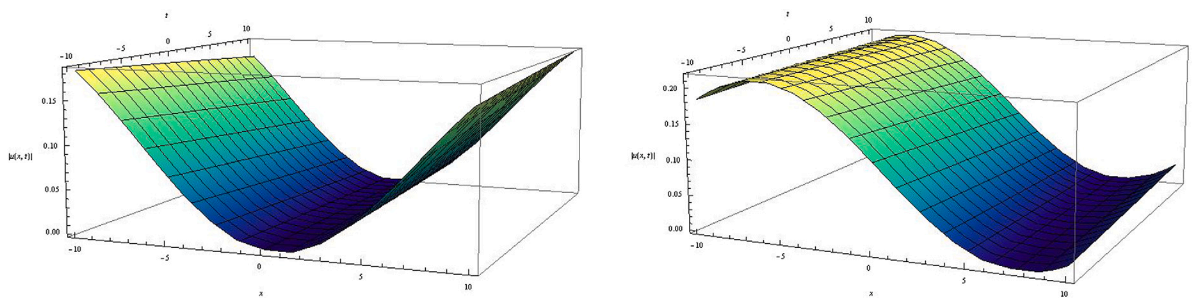

Fig. 5. The graphics of soliton wave solutions $u(x,t)$ of Eqs. (40) and (41) with $a = 5,\ \sigma = 1,\ \omega = 0.3,\ \Omega = 0.1, \epsilon = 1, q = 5, s = 1$..

of the dark and singular soliton solutions respectively with $a = 2$, $\sigma = 1$, $\omega = 1$, $\Omega = 0.1$. On the other hand, Fig. 2 of the solutions (21) and (22) represent periodic wave solutions with $a = 2$, $\sigma = 1$, $\omega = 0.1$, $\Omega = 0.1$. Indeed, the solutions in Figs. 3–4 shows the shape of the exact soliton-like solution of CH-NLS equation. For convenience, other figures are omitted as they exhibit the same behavior of the above solutions. The exact solutions and figures obtained in this paper gives us a different physical interpretation for the CH-NLS equation.

7. Conclusions

Three different techniques, namely, the new approach of generalized (G'/G)-expansion method, the new mapping method, and the modified simple equation method have been employed to find many exact solutions to CH-NLS equation. These exact solutions include dark soliton, singular soliton, and periodic solutions. Further, the graphs of some solitary wave solutions are presented to express the behavior of our solutions. As far as we know, these exact solutions have not been derived in the literature.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] Camassa R, Holm DD. An integrable shallow water equation with peaked solitons. Phys Rev Lett 1993;71:1661–4.
[2] Johnson RS. On solutions of the Camassa–Holm equation. Proc R Soc Lond Ser A Math Phys Eng Sci 2003;459:1687–708.
[3] Boutet de Monvel A, Kostenko A, Shepelsky D, Teschl G. Long-time asymptotics for teh Camassa–Holm equation. SIAM J Math Anal 2009;41:1559–88.
[4] Constantin A, Gerdjikov V, Ivanov RI. Inverse scattering trans-form for the Camassa–Holm equation. Inverse Problems 2006;22:2197–207.
[5] Alam MN, Akbar MA. Some new exact traveling wave solutions to the simplified MCH equation and the (1 + 1)- dimensional combined KdV–mKdV equations. J Assoc Arab Univ Basic Appl Sci 2015;17:6–13.
[6] Seadawy D, Lu AR, Iqbal M. Construction of new solitary wave solutions of generalized Zakharov–Kuznetsov–BenjaminBona–Mahony and simplified modified form of Camassa–Holm equations. Open Phys 2018;16(1):896–909.
[7] Islam MN, Asaduzzaman M, Ali MS. Exact wave solutions to the simplified modified Camassa–Holm equation in mathematical physics. Aims Math 2019;5(1):26–41.
[8] Arnaudon A. On a Lagrangian reduction and a deformation of completely integrable systems. J Nonlinear Sci 2016;26:1133–60.
[9] Arnaudon A. On a deformation of the nonlinear Schrö equation. J Phys A 2016;49:125202.
[10] Mylonas IK, Ward CB, Kevrekidis PG, Rothos VM, Frantzeskakis DJ. Asymptotic expansions and solitons of the Camassa–Holm nonlinear Schrodinger equation. Phys Lett A 2017;381:3965.
[11] Wazwaz AM, Kaur L. Optical solitons for nonlinear Schrodinger (NLS) equation in normal dispersive regimes. Optik 2019;184:428–35.
[12] Wazwaz AM, Kaur L. Optical solitons for nonlinear Schrödinger (NLS) equation in normal dispersive regimes. Optik 2019;184:428–35.
[13] Yang X, Huo D, Hong X. Periodic transmission and control of optical solitons in optical fibers. Optik 2020;216:164752.
[14] Savaissou N, Gambo B, Rezazadeh H, et al. Exact optical solitons to the perturbed nonlinear Schrodinger equation with dual-power law of nonlinearity. Opt Quant Electron 2020;52:318.
[15] EL-Wakil SA, Abdou MA. New exact traveling wave solutions using modified extended tanh-function method. Chaos Solitons Fractals 2007;31:840–52.
[16] Fan E. Extended tanh-function method and its applications to nonlinear equations. Phys Lett A 2000;277:212–8.
[17] Wazwaz AM. Exact solutions to the double sinh-Gordon equation by the tanh method and a variable separated ODE. method. Comput Math Appl 2005;50:1685–96.
[18] Wazwaz AM. A sine-cosine method for handling nonlinear wave equations. Math Comput Modelling 2004;40:499–508.
[19] Fan E, Zhang H. A note on the homogeneous balance method. Phys Lett A 1998;246:403–6.
[20] Adomian G. Solving frontier problems of physics: The decomposition method. Boston: Kluwer Academic Publishers; 1994.
[21] Mathanaranjan T, Himalini K. Analytical solutions of the time-fractional nonlinear Schrodinger equation with zero and non zero trapping potential through the Sumudu Decomposition method. J Sci Univ Kelaniya 2019;12:21–33.
[22] Dai CQ, Zhang JF. Jacobian elliptic function method for nonlinear differential difference equations. Chaos Solutions Fractals 2006;27:1042–9.
[23] Fan E, Zhang J. Applications of the Jacobi elliptic function method to special-type nonlinear equations. Phys Lett A 2002;305:383–92.
[24] Abdou MA. The extended F-expansion method and its application for a class of nonlinear evolution equations. Chaos Solitons Fractals 2007;31:95–104.
[25] Zhang JL, Wang ML, Wang YM, Fang ZD. The improved F-expansion method and its applications. Phys Lett A 2006;350:103–9.

[26] He JH, Wu XH. Exp-function method for nonlinear wave equations. Chaos Solitons Fractals 2006;30:700–8.
[27] Aminikhad H, Moosaei H, Hajipour M. Exact solutions for nonlinear partial differential equations via Exp-function method. Numer Methods Partial Differential Equations 2009;26:1427–33.
[28] Wang ML, Zhang JL, Li XZ. The (G'/G)- expansion method and traveling wave solutions of nonlinear evolutions equations in mathematical physics. Phys Lett A 2008;372:417–23.
[29] Zhang S, Tong JL, Wang W. A generalized (G'/G)- expansion method for the mKdv equation with variable coeffcients. Phys Lett A 2008;372:2254–7.
[30] Zayed EME, Gepreel KA. The (G'/G)- expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics. J Math Phys 2009;50:013502–13.
[31] Zahran EHM, Khater Mostafa MA. Exact solution to some nonlinear evolution equations by The (G'/G)- expansion method. Jökull J 2014;64:5.
[32] Naher H, Abdullah FA. New approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method for nonlinear evolution equation. AIP Adv 2013;3(3):032116.
[33] Naher H. New approach of (G'/G)-expansion method and new approach of generalized (G'/G)-expansion method for ZKBBM equation. J Egypt Math Soc 2015;23:42–8.
[34] Zeng X, Yong X. A new mapping method and its applications to nonlinear partial differential equations. Phys Lett A 2008;372:6602–7.
[35] Zayed EME, Al-Nowehy A-G. Solitons and other exact solutions for a class of nonlinear Schrö-type equations. Optik - Int J Light Electron Opt 2017;130:1295–311.
[36] Jawad AJM, Petkovic MD, Biswas A. Modified simple equation method for nonlinear evolution equations. Appl Math Comput 2010;217:869–77.
[37] Zayed EME, Hoda Ibrahim SA. Exact solutions of nonlinear evolution equation in mathematical physics using the modified simple equation method. Chin Phys Lett 2012;29:060201–4.
[38] Zayed EME, Arnous AH. Exact solutions of the nonlinear ZK-MEW and the potential YTSF equations using the modified simple equation method. AIP Conf Proc 2012;1479:2044–8.
[39] Zahran Emad HM, Khater Mostafa MA. The modified simple equation method and its applications for solving some nonlinear evolutions equations in mathematical physics. Jokull 2014;64:5.
[40] Liu SZ, Zhou Q, Biswas A, Liu W. Phase-shift controlling of three solitons in dispersion-decreasing fibers. Nonlinear Dynam 2019;98:395–401.
[41] Wazwaz AM. Solitary wave solutions of the generalized shallow water wave (GSWW) equation by Hirota's method, tanh-coth method and Exp-function method. Appl Math Comput 2008;202:275–86.
[42] Deng SF, Qin ZY. Darboux transformation and its set application in soliton theory. Shanghai: Shanghai Science and Technology Education Press; 1999.
[43] Guan X, Liu WJ, Zhou Q, Biswas A. Darboux and Backlund transformations for the nonisospectral KP equation. Phys Lett A 2006;357:467–74.
[44] Yang C, Li W, Yu W, Liu M, Zhang Y, Ma G, et al. Amplification, reshaping, fission and annihilation of optical solitons in dispersion-decreasing fiber. Nonlinear Dynam 2018;92:203–13.
[45] Chen J, Luan Z, Zhou Q, et al. Periodic soliton interactions for higher-order nonlinear Schrödinger equation in optical fibers. Nonlinear Dyn 2020;100:2817–21.