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Mathematical Model to Predict the Compressive Strength of Pervious Concrete

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K e y w o r d s

Pervious concrete
Aggregate
Aggregate to cement ratio
Compaction energy
Compressive strength

A B S T R A C T

This study examined the impact of aggregate size, aggregate-to-cement ratio and compaction energy on the compressive strength of pervious concrete and proposed a mathematical model to predict the compressive strength. Two sets of aggregate sizes (5-12 mm, 12-18 mm), five sets of aggregate-to-cement ratio (3.0, 3.5, 4.0, 4.5 and 5.0) and seven sets of compaction energy (0, 15, 30, 45, 60, 75 and 90 blows) were deployed in this study. Pervious concrete was cast with a total of 70 different mix designs (210 cubes) and tested for compressive strength. From the analysis of test data, mathematical connections between compressive strength and aggregate size, aggregate-to-cement ratio, and compaction energy were developed. Compressive strength was accurately represented by the established models in terms of aggregate size, A/C ratio, and compaction energy. These models would be helpful to practitioners and researchers when setting pervious concrete mix design parameters for a range of pervious concrete applications.

1. INTRODUCTION

Pervious concrete is a composite material comprising cement, coarse aggregate and water. It is very permeable, making it an environmentally friendly material that has been in high demand for the past 40 years throughout the whole world (Aamer Rafique Bhutta *et al*., 2013). In many applications such as parking lots, pervious concrete is used to drain and permeate water, reducing surface runoff and enhancing the environmental performance of the existing site by increasing base flow, maintaining water quality, reducing flooding around the site, and reserving parking spaces for the property owner. Additionally advantageous, it prevents surface flashing and glistening at night, which increases driver comfort and safety while also maintaining a peaceful atmosphere. In addition, the pores of pervious concrete may accumulate heat, changing the earth's surface's humidity and temperature (Yang and Jiang, 2003).

Porosity, permeability and compressive strength are considered as major parameters for the recommendation of pervious concrete for its application. Porosity is a result of pores, which are the hollow spaces between aggregates covered with cement paste. The ratio of the volume of voids to the combined volume of the mix is called porosity.

The properties of pervious concrete depend on aggregate grading and shape, Aggregate to Cement (A/C) ratio, Water to Cement (W/C) ratio and compaction energy. Generally, the mix design of pervious concrete aims to derive a balance between workability properties, porosity and strength (Debnath and Sarkar, 2020). Studies recommend that the aggregate size to be used in pervious concrete lies in the range of 4.75 mm to 19.5 mm to retain a considerable amount of air voids in the mix (Debnath and Sarkar, 2020, Maguesvari and Narasimha, 2013, Chandrappa and Biligiri, 2016). A variety of A/C ratios from 2.0 to 10.0 have been adopted in pervious concrete (Chandrappa and Biligiri, 2016, Zhong *et al*., 2018, Mohammed *et al*., 2016, Deo and Neithalath, 2011). An increase in A/C ratios results in low strengths as a result of inadequate amount of cement paste to bind aggregates together, while low A/C ratios produce a substantial quantity of cement paste to fill up the pores in the aggregate skeleton that leads to a reduction in permeability (Debnath and Sarkar, 2020).

Reviewed literature displayed a wide range of works that attempted to formulate a relationship between the above said parameters and the mix design. Maguesvari *et al*. (2013) showed that the strength characteristics of pervious concrete improve with the increase in fine aggregate content with a compromise in permeability. Neithalath *et al*. (2010) modelled the pore size distribution in pervious concrete as a probability density function and compared it for various aggregate grading. The compaction aspects were explored in the literature as to how they influence the strength and permeability characteristics of pervious concrete (Lian and Zhuge, 2010, Zhuge, 2008, Deo and Neithalath, 2011). To the best of the author's knowledge, the prime focus of the majority of the existing studies was to derive distinct relationships between strength, porosity, A/C ratio, and compaction energy. However, the combined effect of pervious concrete parameters remains the least explored. Since compressive strength is a vital parameter in pervious concrete mix designs, it is inevitable to understand its behaviour with respect to aggregate size, A/C ratio and compaction energy, which in turn can be correlated to compressive strength. The presented work in this paper intends to form a mathematical relationship to determine the compressive strength of previous concrete using aggregate size, A/C ratio and compaction energy through an experimental study.

2. MATERIALS AND METHODS

As the main binding material, ordinary Portland cement (OPC) was utilized. The natural granite-gneisses rock of igneous origin was used in this study. In order to cast the pervious concrete, the aggregate was carefully sieved down to a size of 5 to 18 mm and separated into two categories as the size ranges between 5-12 mm and 12-18 mm. The investigated aggregate qualities are listed in Table 1, and the test results show that the aggregates are of sufficient quality to be utilized as coarse aggregates in the casting of pervious concrete.

Table 1 Mechanical properties of aggregate.

Properties	Value
Bulk density (kg/m^3)	1491
Specific gravity	2.72
Absorption (%)	0.20
Moisture content (%)	0.35
Aggregate Impact value - AIV (%)	25.5
Aggregate Crushing Value - ACV (%)	30.5
Flakiness Index	14.7
Elongation Index (%)	24.02

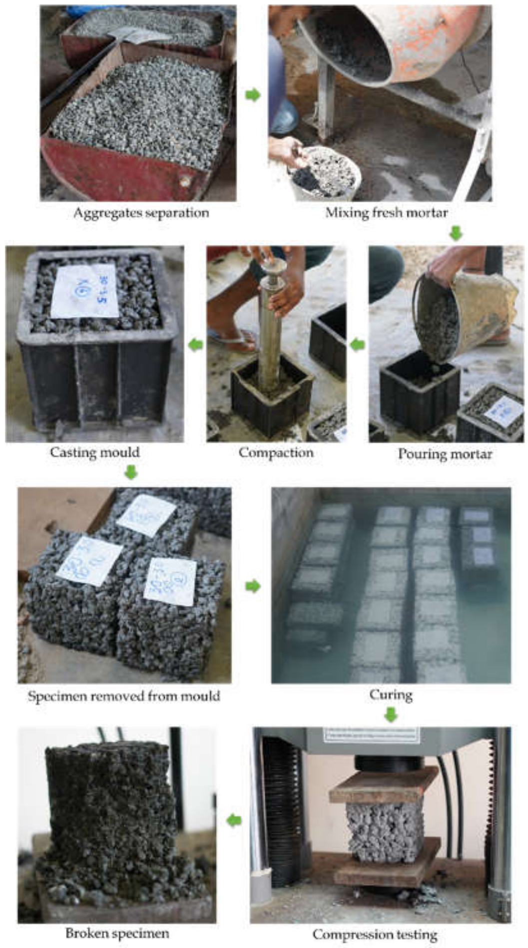

Figure 1 Specimen preparation and testing.

Table 2 provides information on the mix ratio of components used in pervious concrete mortar. For all mixtures, a water-to-binder ratio of 0.3 is used. For testing, cubes of 150 mm x 150 mm x 150 mm were cast. Three layers of compaction were applied, with a total of 0, 15, 30, 45, 60, 75, and 90 blows with a standard proctor hammer. A wooden board was put on top of the mould and softly tamped with a tamping rod to create an even surface on the sample.

Table 2 Mix design for pervious concrete.

Mix-ID	A/C ratio	Cement (kg)	Aggregates (kg) 5-12 mm	Aggregates (kg) 12-18 mm	Water (l)
S-3.0	3.0	379.7	1139.2		113.9
S-3.5	3.5	368.3	1289.0		110.5
S-4.0	4.0	327.5	1309.9		98.2
S-4.5	4.5	291.2	1310.3		87.4
S-5.0	5.0	265.5	1327.5		79.6
L-3.0	3.0	419.4		1258.1	125.8
L-3.5	3.5	361.7		1266.1	108.5
L-4.0	4.0	317.5		1270.1	95.3
L-4.5	4.5	290.7		1308.3	87.2
L-5.0	5.0	261.3		1306.3	78.4

According to ASTM-C109 (2020), the compressive strengths of cured concrete cubes were tested. Three cubes for each mortar mix composition were evaluated using a Universal Testing Machine for compressive strength after curing for 28 days (UTM). A loading rate of 1 mm/min was used to apply the axial load.

3. RESULTS AND DISCUSSION

3.1. Experimental Results

Figure 2 illustrates the compressive strength variation with compaction energy and A/C ratios for aggregate sizes 5-12 mm and 12-18 mm, respectively. For a particular A/C ratio, compressive strength values show a gradual increase with the number of blows up to 30 to 45 and no significant change thereafter. Compaction has increased the packing density in aggregates and hence enhanced the ability of the mix to withstand higher compressive stresses. Low A/C ratios exhibited significant differences in strengths for varying compaction efforts as high as 23 MPa between 0 and 60 blows. At low A/C ratios, surplus cement pastes induced slaking, have predominantly affected the packing of aggregates. Low compaction effort was insufficient to overcome the slaking effect. However, when the number of blows were increased, additional compaction provided sufficient energy to the aggregates to float in the cement paste matrix and to form a firm packing. This phenomenon produced vast differences in compressive strengths for low A/C ratio mixes subjected to varying compaction efforts. For a higher A/C ratio, the compressive strengths of pervious concrete for varying compaction effort spanned a small range as the slaking effect due to cement paste became trivial and hence the compaction only improved aggregate packing. The mere aggregate packing was inadequate to provide noticeable changes in compressive strengths.

As expected, the compressive strength of previous concrete was reduced with an increase in the A/C ratio. With the subsequent increase in the A/C ratio, the mixes suffered inadequate cement paste to coat around the aggregate, resulting in the loss of strength. For higher A/C ratios, the cement paste formed a thin layer of bonding between aggregates that failed to withstand stresses and thus induced premature failure in pervious concrete mix. When the A/C ratio increases, the PC becomes almost like an ordinary aggregate packing with inadequate binding of particles. Researchers have adequately explored the influence of cement paste on the mechanical properties of previous concrete and recommended optimizing it (Ryshkewitch, 1953, Yahia and Kabagire, 2014).

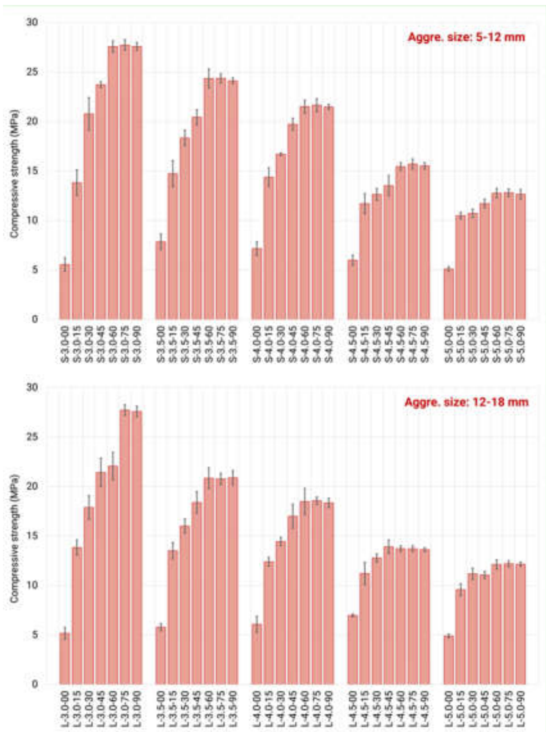

Figure 2 Compressive strength variation with A/C ratio and compaction energy for aggregate size 5-12 mm (top) and 12-18 mm (bottom).

When two aggregate sizes were compared, pervious concrete cast with smaller aggregates showed slightly better compressive strength compared to the one cast with larger aggregates. Generally, the compressive strength ratio between pervious concrete with larger aggregates and finer aggregate was increased with an A/C ratio and its lies between 0.85-0.97.

It can be concluded that the combined influence of aggregate size, A/C ratio and compaction energy on compressive strength is predominant for A/C ratios from 3.0 to 4.0, and minor for A/C ratios above 4.0. These comparisons emphasized the necessity of developing a common model to combine the effects of aggregate size, compaction effort and A/C ratio on compressive strength.

3.2. Statical Analysis

In order to determine the impact of an independent variable (aggregate size, A/C ratio, and compaction energy) on the compressive strength of pervious concrete, an analysis of variance (ANOVA) was performed. The results are shown in Table 3.

Table 3 Statical analysis on compaction energy on compressive strength of pervious concrete.

	Change in Blows	A/C ratio 3.0	3.5	4.0	4.5	5.0
Aggregate size: 5-12 mm	0-15	√ (0.001)	√ (0.003)	√ (0.001)	√ (0.002)	√ (0.000)
	15-30	√ (0.010)	√ (0.029)	√ (0.027)	X (0.326)	X (0.579)
	30-45	X (0.068)	√ (0.050)	√ (0.002)	X (0.346)	X (0.078)
	45-60	√ (0.029)	√ (0.011)	√ (0.050)	X (0.067)	X (0.082)
	60-75	X (0.801)	X (0.973)	X (0.840)	X (0.612)	X (0.931)
	75-90	X (0.786)	X (0.729)	X (0.739)	X (0.699)	X (0.753)
Aggregate size: 12 -18 mm	0-15	√ (0.000)	√ (0.000)	√ (0.001)	√ (0.006)	√ (0.000)
	15-30	√ (0.016)	√ (0.032)	√ (0.010)	X (0.136)	X (0.052)
	30-45	X (0.057)	X (0.063)	X (0.049)	X (0.117)	X (0.829)
	45-60	X (0.675)	X (0.082)	X (0.308)	X (0.726)	X (0.061)
	60-75	X (0.705)	X (0.931)	X (0.933)	X (0.983)	X (0.866)
	75-90	X (0.891)	X (0.862)	X (0.630)	X (0.757)	X (0.866)

Additionally, SPSS software was used to analyze the variation in data between two subsequent compaction energies. A significance level (α) of 0.05 was fixed for this test. For finer aggregates (5-12 mm), compaction energy had a statistically significant effect on compressive strength until 60 blows for A/C ratios 3.0, 3.5 and 4.0. But with a higher A/C ratio (4.5 and 5.0), compaction energy had a statistically significant effect on compressive strength only until 15 blows and there was no notable change in compressive strength afterwards. A similar trend was observed for the A/C ratio of 4.5 and 5.0 with large aggregates (12-18 mm). But for large aggregates, compaction energy had a statistically significant effect on compressive strength until 30 blows for A/C ratios 3.0, 3.5 and 4.0.

Table 4 Statical analysis on aggregate size on compressive strength of pervious concrete.

No of Blows	A/C ratio 3.0	3.5	4.0	4.5	5.0
0	X (0.583)	√ (0.029)	X (0.222)	X (0.060)	X (0.382)
15	X (0.989)	X (0.329)	X (0.055)	X (0.658)	X (0.131)
30	X (0.116)	√ (0.036)	√ (0.002)	X (0.798)	X (0.419)
45	√ (0.092)	√ (0.094)	√ (0.048)	X (0.682)	X (0.170)
60	√ (0.007)	√ (0.025)	√ (0.045)	√ (0.008)	X (0.245)
75	√ (0.000)	√ (0.002)	√ (0.005)	√ (0.010)	X (0.152)
90	√ (0.000)	√ (0.016)	√ (0.001)	√ (0.002)	X (0.241)

Table 4 summarizes the significance of the aggregate size effect on compressive strength for various A/C ratios and compaction energy. Aggregate size effect was not observed for the number of blows less than 30 for all A/C ratio. For higher blows (45 and above) and a low A/C ratio (less than 4.5), the compressive strength is significantly affected by aggregate size.

3.3. Mathematical Model for Individual Aggregate Size

According to Figure 3, the rate of change in compressive strength with respect to compaction energy first exhibited a rising trend before progressively achieving an optimum strength with compaction energy. Therefore, a saturation curve might be used to illustrate the variation in compressive strength of pervious concrete and compaction energy for a certain A/C ratio as shown in Eq. (1) (Anburuvel and Subramaniam, 2022).

$$\sigma = \frac{a}{1+be^{(-cB)}} \quad (1)$$

where B stands for compaction given by the number of blows and σ stands for compressive strength (in MPa). Model parameters a, b, and c represent the range of compressive strength variation throughout compaction attempts, the effect of initial compaction, and the rate of change in compressive strength.

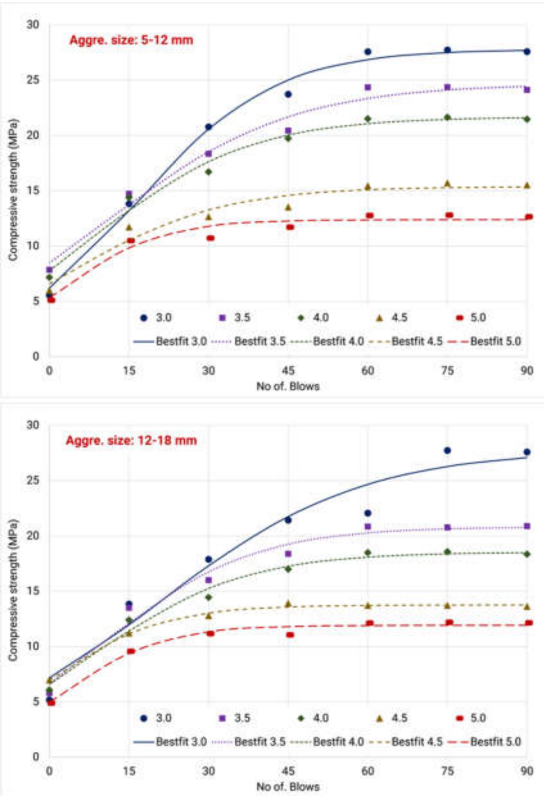

Figure 3 Rate of change in compressive strength with respect to compaction energy.

Using best-fit curve fitting, the value of a, b and c for every A/C ratio and aggregate size was determined and summarized in Table 5.

Compressive strength, responding to the change in A/C ratio, varied for the same compaction effort. By incorporating the effect of aggregate gradation in parameters a, b and c simultaneously, a general model could be developed. The estimated parameters a, b and c were thereafter mapped with corresponding A/C ratios to derive a common relationship in terms of A/C ratio for each aggregate size. Given that AC refers to the A/C ratio, derived parameters; a, b and c and best-fit equations are given in Table 6.

Table 5 Statical analysis of compaction energy on compressive strength.

Aggregate Size	A/C ratio	a	b	c
5-12 mm	3.0	27.7923	3.4899	0.0765
	3.5	24.6943	1.9182	0.0583
	4.0	21.6778	1.7921	0.0684
	4.5	15.3781	1.3383	0.0708
	5.0	12.3822	1.3213	0.1083
12-18 mm	3.0	27.8172	2.8896	0.0519
	3.5	20.8217	2.1378	0.0724
	4.0	18.5390	1.8194	0.0710
	4.5	13.7591	0.9705	0.0944
	5.0	11.9167	1.3956	0.1093
	3.0	27.8172	2.8896	0.0519

Table 6 The estimated parameters a, b and c.

Parameter	5-12 mm	12-18 mm
a	$-8.027AC + 52.494$	$-7.773AC + 49.662$
b	$0.795AC^2 - 7.342AC + 18.225$	$0.521AC^2 - 4.999AC + 13.241$
c	$0.029AC^2 - 0.222AC + 0.475$	$0.004AC^2 + 0.004AC - 0.031$

By combining Eq. (1) and the equation given in Table 6, a general relationship among compressive strength, A/C ratio and compaction energy can be derived as follows.

For aggregate size: 5-12 mm:

$$\sigma = \frac{-8.027AC + 52.494}{1 + (0.795AC^2 - 7.342AC + 18.225)e^{(-0.029AC^2 + 0.222AC - 0.475)B}} \quad (2)$$

R^2 = 0.9865, RMSE = 1.052 MPa, MARE = 6.16%

For aggregate size: 12-18 mm:

$$\sigma = \frac{-7.773AC + 49.662}{1 + (0.521AC^2 - 4.999AC + 13.241)e^{(-0.004AC^2 - 0.004AC + 0.031)B}} \quad (3)$$

R^2 = 0.9826, RMSE = 1.199 MPa, MARE = 7.96%

where R^2 – Coefficient of Determination, RMSE - Root Mean Square Error, and MARE - Mean Absolute Relative Error.

The proposed relationship was verified with a set of data of measured compressive strengths for various A/C ratios and compaction energy in this study. Figure 4 illustrates the graphical comparison of the measured and estimated compressive strengths of PC. The values are concentrated near the line of best fit, which implies that the proposed model performs well.

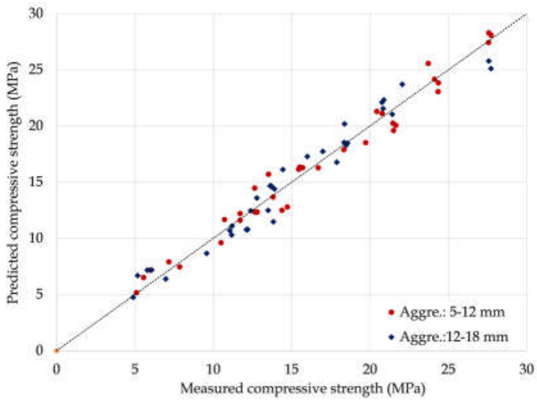

Figure 4 Predicted vs. measured compressive strength of pervious concrete for individual aggregate sizes.

3.4. Mathematical Model Including Aggregate Size

To the predicted equation, aggregate size was also incorporated in addition to the A/C ratio to map the parameters a, b and c in Table 5. Therefore, model parameters a, b and c were mapped with respective aggregate size (S) and A/C ratio (AC) to derive a generalised equation as follows.

- a = 54.3574 - 0.2791S - 7.9AC
- b = 28.6349S-0.1169 AC-1.8198
- c = 0.023S0.0573 1.3027AC

By combining the above equations and Eq (1), a general relationship among compressive strength, aggregate size, A/C ratio and compaction energy can be derived as follows.

$$\sigma = \frac{54.3574 - 0.2791S - 7.9AC}{1 + (28.6349S^{-0.1169}AC^{-1.8198})e^{(-0.023S^{0.0573}1.3027^{AC})*B}} \quad (4)$$

$R^2 = 0.9821$, RMSE = 1.151 MPa, MARE = 7.29%

The proposed relationship was verified with a set of data of measured compressive strengths for various aggregate sizes, A/C ratios and compaction energy in this study. Figure 5 illustrates the graphical comparison of the measured and estimated compressive strengths of PC. The values are concentrated near the line of best fit, which implies that the proposed model performs well.

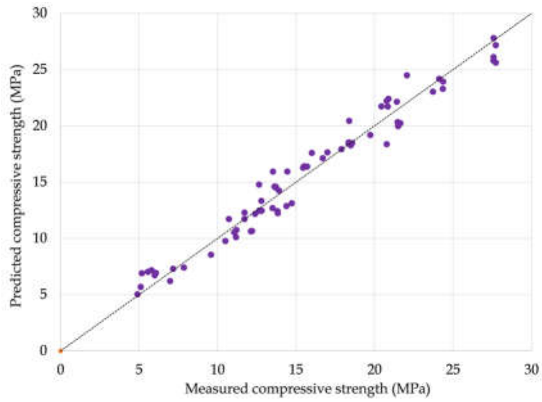

Figure 5 Predicted vs. measured compressive strength of pervious concrete.

3.5. Sensitivity Analysis

A sensitivity analysis was performed to assess the influence of the aggregate size, A/C ratio and compaction energy variability on the compressive strength of pervious concrete. The range of some of these mix design parameters highly varies based on the design requirements of pervious concrete. Thus, the main objective of this sensitivity analysis was to assess how the variability of these mix design parameters can affect the compressive strength of pervious concrete. The sensitivity analysis was performed by changing parameters, as summarized below.

- Aggregate size: range 2.5 – 25 mm, mean of 9.5 mm
- A/C ratio: 3.0 – 6.0, mean of 4.5
- Blows: 0 – 90, mean of 30

The influence of aggregate size, A/C ratio and compaction energy on compressive strength is visualized in Figure 6, where the non-dimensional compressive strength (normalized by the compressive strength calculated using the mean value of mix design parameters – fs/fs,ref) is compared with the non-dimensional mix design parameters tested in the sensitivity analysis (normalized by the respective parameter mean value – x/xref). The A/C ratio has the greatest influence, especially when it is increased. It was followed by compaction energy, especially until it reaches 30 blows. Compared with the A/C ratio and compaction energy, the aggregate size has less influence on the compressive strength of pervious concrete.

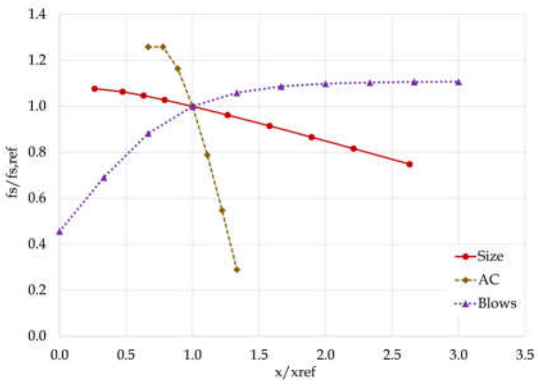

Figure 6 Sensitivity analysis.

4. CONCLUSIONS

This study examined the impact of aggregate size, A/C ratio and compaction energy on compressive strength of pervious concrete and proposed mathematical model to predict the compressive strength. Based on the results of this study, the following models were developed to predict compressive strength (σ) in terms of aggregate size (S), A/C ratio (AC) and compaction energy (B).

For aggregate size: 5-12 mm:

$$\sigma = \frac{-8.027AC + 52.494}{1 + (0.795AC^2 - 7.342AC + 18.225)e^{(-0.029AC^2 + 0.222AC - 0.475)B}}$$

For aggregate size: 12-18 mm:

$$\sigma = \frac{-7.773AC + 49.662}{1 + (0.521AC^2 - 4.999AC + 13.241)e^{(-0.004AC^2 - 0.004AC + 0.031)B}}$$

For model including aggregate size:

$$\sigma = \frac{54.3574 - 0.2791S - 7.9AC}{1 + (28.6349S^{-0.1169}AC^{-1.8198})e^{(-0.023S^{0.0573}1.3027^{AC})*B}}$$

The developed models predicted the compressive strength of pervious concrete mixes made with aggregates in the size range of 5-12 mm, 12-18 mm and all range at uncertainties of 6.2%, 8.0% and 7.3% of the mean values, respectively.

Compressive strength is accurately represented by the established models in terms of aggregate size, A/C ratio, and compaction energy. These models would be helpful to practitioners and researchers when setting pervious concrete mix design parameters for a range of pervious concrete applications. The research that has been provided so far may be expanded in the future to examine how these factors affect the permeability and porous characteristics of pervious concrete.

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